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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01

Paper 1

General comments

The paper proved to be very accessible for the majority of candidates and there were relatively few scripts from candidates who should not have been entered for the examination. Apart from the term 'unit vector' required in **Question 8**, candidates showed good understanding of all parts of the syllabus. Presentation was generally of a pleasing standard.

Comments on specific questions

Question 1

This proved to be a reasonable starting question for most candidates and approximately half of all attempts were correct. Although most candidates recognised the term in x, there were many misunderstandings about the use of the binomial expansion, most seriously the lack of inclusion of binomial coefficients. $(3x)^3$ was often replaced by $3x^3$ and $(-2)^2$ often appeared as either 2 or -4 with a final answer of -1080 being particularly common. Weaker candidates used the notation $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$, but showed a complete lack of understanding by replacing this by 2.5.

Answer: 1080.

Question 2

(i) This proved to be an easy question for most candidates, though occasionally there was confusion between arithmetic and geometric progressions. Occasionally r was given as $\frac{3}{2}$ instead of $\frac{2}{3}$, but knowledge of the formula for the sum of ten terms was sound and the answer was usually correct. Premature approximation of r to 0.6 or 0.7 was a common error that lost the last accuracy mark.

(ii) Knowledge of the formulae required was good, but unfortunately a large proportion failed to realise the need to find the number of terms in the progression. Correct use of $a + (n-1)d$ led to $n = 32$, but it was very common to see n calculated as 31 or taken as some other value (180 or 25 being the usual offerings). Use of $d = +5$ instead of -5 was another common error.

Answers: **(i)** 239; **(ii)** 3280.

Question 3

The majority of candidates recognised that the shaded area could be calculated directly by subtracting the area of a sector from the area of a right-angled triangle. Finding the area of the sector in terms of π presented few problems and most candidates realised the need to use trigonometry to find the area of the triangle. Obtaining a correct decimal answer presented few problems – obtaining answers in terms of $\sqrt{3}$ proved to be more difficult. Only a minority of candidates showed confidence in being able to use the surd form for $\sin\left(\frac{1}{3}\pi\right)$ or $\tan\left(\frac{1}{3}\pi\right)$ correctly. There was also confusion with some weaker candidates of the fact that the angle was given in radians.

Answer: $18\sqrt{3} - 6\pi$ cm^2.

Question 4

(i) Although there were some excellent solutions to the question, candidates generally showed lack of ability in sketching trigonometrical graphs. Many automatically sketched graphs in the range 0 to 2π instead of 0 to π, though this lost time rather than marks. Many others preferred to draw accurate graphs, and again this was penalised only in time. Many weaker candidates failed completely to recognise the difference between the sketches of $y = 2\sin x$ and $y = \sin 2x$ and similarly between $y = 2\cos x$ and $y = \cos 2x$.

(ii) The majority of candidates failed to realise the link between this part of the question and the sketches drawn in part **(i)**. Many ignored the word 'hence' and attempted to solve the equations by various methods. The majority of these candidates gave the solutions to the equation rather than the number of solutions, thereby gaining no credit. The failure to recognise that the number of solutions was the same as the number of intersections of the two graphs was surprising.

Answers: **(i)** Sketches; **(ii)** 2.

Question 5

This was well answered and there were many completely correct solutions.

(i) Candidates showed pleasing manipulative skills in forming and solving a correct quadratic equation in either x or y and it was rare for candidates not to obtain the coordinates of M.

(ii) This part presented more problems with many candidates failing to realise the need to use calculus to find the gradient of the tangent. It was also common to see $2x - 4$ equated with either 0 (as at a stationary point) or with $9 - 3x$ (the expression for y).

(iii) Most candidates obtained this last mark, which was a follow through mark for their answer for Q. Surprisingly, very few candidates realised that the answer to the distance between (0.5, 7.5) and (0.5, 5.25) could be evaluated directly without the need for the formula for the distance between two points.

Answers: **(ii)** Q(0.5, 5.25); **(iii)** 2.25.

Question 6

(i) This was well answered with most candidates realising the need to replace $\cos^2 x$ by $1 - \sin^2 x$.

(ii) Virtually all candidates realised the need to use part **(i)** to obtain a quadratic equation for $\sin x$. There were however errors in factorising the quadratic and the more serious error of solving $7\sin x - 2\sin^2 x = 3$ as $\sin x = 3$ or $7 - 2\sin x = 3$ was often seen from weaker candidates. Obtaining answers to $\sin^{-1}\left(\frac{1}{2}\right)$ in radians presented difficulty with a significant number of attempts being left in degrees.

(iii) This proved to be the most difficult question on the paper with only a handful of candidates realising that the minimum value of f occurred when $\sin^2 x = 0$ and the maximum occurred when $\sin^2 x = 1$.

Answers: **(i)** $a = 3$, $b = 2$; **(ii)** 0.524, 2.62; **(iii)** $3 \leqslant \mathrm{f} \leqslant 5$.

Question 7

The question as a whole was poorly answered with confusion over the difference between the equation of the curve and the equation of the tangent or normal.

(i) Many candidates failed to realise that there was no need to either integrate or differentiate. Substitution of x = 3 led directly to the gradient of the tangent being 2 and consequently that the gradient of the normal was $-\frac{1}{2}$. Many candidates attempted to integrate to find an expression for y; some even followed this by differentiation to return to the given expression for $\frac{dy}{dx}$. There were many solutions seen in which the gradient of the normal was left in algebraic form. Several candidates lost the last mark through failure to express the equation of the line in the required form i.e. $ax + by = c$.

(ii) When candidates realised the need to integrate, the standard of integration was generally good though omission of $\frac{1}{4}$ (division by the differential of $4x - 3$) was a common error. Many candidates failed to realise that $\frac{1}{\sqrt{4x-3}}$ was $(4x - 3)^{-\frac{1}{2}}$. Many attempts failed to realise the need to find the constant of integration.

Answers: **(i)** $x + 2y = 9$; **(ii)** $y = 3\sqrt{4x-3} - 6$.

Question 8

(i) This was very well answered with the majority of attempts being correct. The most common source of lost marks occurred in the final answer mark when the angle was expressed in degrees.

(ii) This was badly answered with most candidates failing to recognise the meaning of 'unit vector'. The manipulation of vectors required to obtain an expression for vector $\overrightarrow{OC}$ caused problems with such errors as $\overrightarrow{AB}$ = **a** − **b** or **a + b** being common. Many candidates failed to realise that $\overrightarrow{OC}$ **= b +** $\frac{1}{2}$**(b – a)**. Even when $\overrightarrow{OC}$ was correctly evaluated, it was very rare to see this divided by the modulus of the vector to obtain the unit vector.

Answers: **(i)** 0.907 radians; **(ii)** $\frac{1}{12}(-8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$.

Question 9

This proved to be a source of high marks for most candidates who showed a very good understanding of functions.

(i) The vast majority obtained a correct expression for ff(x) and it was rare to see the error of taking this as $[\mathrm{f}(x)]^2$.

(ii) Most candidates obtained a correct quadratic equation for f(x) = g(x) and it was pleasing to see the vast majority recognising the need to look at the sign of '$b^2 - 4ac$'. Although most set this to 0, there were many where it was taken as >0 or <0.

(iii) Candidates showed a very good understanding of 'completing the square'.

(iv) When a similar question was set a couple of years ago, this technique presented a lot of problems. This time however, the solutions were pleasing with most candidates realising the need to use the expression obtained in part **(iii)** to express x in terms of y and then to replace y by x. There were however, only a few solutions in which candidates realised that the domain of h^{-1} was equal to the range of h.

Answers: **(i)** x = 5; **(ii)** a = 16; **(iii)** p = 3 and q = 9; **(iv)** $\mathrm{h}^{-1}(x) = \sqrt{(x + 9)} + 3$, $x \geqslant -9$.

Question 10

This question was well answered and a source of high marks. The standard of differentiation and integration was good.

(i) Apart from a few candidates who took $\frac{2}{x}$ as $2x^{-\frac{1}{2}}$ or as $2x^{\frac{1}{2}}$, the differentiation was generally accurate.

(ii) The algebra required to solve $2x - \frac{2}{x^2} = 0$ was well done and most candidates obtained the point (1, 3). Most also looked at the sign of the second differential and made a correct deduction about the nature of the turning point. Very few candidates used the alternative methods of looking at the sign of the first differential around the stationary point or at looking at values of y around the stationary point.

(iii) Whilst there were many correct solutions to this part, there were many poor attempts. Many took the volume as $\int y\,dx$, others omitted the π from the formula, whilst others assumed $(a + b)^2$ to be $a^2 + b^2$. The integration was generally accurate and most candidates used the limits 1 to 2 correctly.

Answers: **(i)** $2x - \frac{2}{x^2}$, $2 + \frac{4}{x^3}$ **(ii)** (1, 3), Minimum point; **(iii)** 14.2π or 44.6.

Paper 9709/02

Paper 2

General comments

Many candidates proved themselves well prepared for the examination and most could make good attempts at several questions. However, there were a very significant number of extremely weak candidates who had great difficulty addressing even the most basic tests of key items in the syllabus.

Questions were usually addressed sequentially, though weaker attempts often covered three or four separate calculations. Thos questions that were generally well answered were **Questions 3**, **4**, **6 (iii)**, and **8 (iii)**. Severe difficulties were experienced in **Questions 2**, **6 (i)**, **7 (iii)** and **8 (iv)**.

Although the Examiners were pleased at substantial improvements in certain areas, e.g. **Question 6 (ii)** and **(iv)**, certain key operations still seem poorly understood by many candidates, e.g. how to differentiate a product of two functions (see **Question 5**) and the basic rules of differentiation and integration (see **Question 7 (ii)** and **(iii)** and **Question 8 (iii)** and **(iv)**). The difference between obtaining a solid mark for this examination and a poor one is usually dependent on a candidate's ability to differentiate and integrate, and the Examiners urge Centres to concentrate efforts into fully preparing their candidates in these areas.

Comments on specific questions

Question 1

Those candidates (a majority) who squared each side of the inequality generally faired very well. The Examiners were especially pleased that former frequent errors such as squaring on *one* side only, or producing only 2 terms when squaring the left-hand side, have all but disappeared. There remains confusion over the effect of multiplying through an inequality by -1. A good rule, if in doubt, is to choose a specific value of x as a check; here, if uncertain as to whether $x > -\frac{1}{2}$ or $x < -\frac{1}{2}$, the value $x = 0$ satisfies the initial inequality, and so must belong to the set of values forming the final solution.

Candidates who used a more simplistic method, e.g. taking 4 cases, $x + 1 > x$, $x + 1 > -x$, $x + 1 < x$, $x + 1 < -x$, rarely got near to the final result.

Answer: $x > -\frac{1}{2}$.

Question 2

Few candidates could make fruitful progress, despite realising that it was necessary to use logarithms. Almost all solutions foundered on a lack of ability to do the latter properly on the right-hand side. Instead of obtaining ln11 + 3.2ln$x$, most solutions featured 3.2ln11x, or 11 × 3.2ln11x or x × 3.2ln11, for example, thus disqualifying the candidate from scoring any marks. Several candidates successfully obtained the correct solution via the equation $x^{0.7} = 11$, without further working.

Answer: 30.7.

Question 3

Few candidates noted that a solution x = 90° emerges from a common factor cos$x$ on both sides of the equation, having deleted that factor from the generally obtained equation 6sin$x$cos$x$ = cos$x$ or cos$x$(6sin$x$ – 1) = 0. Some attempts were made to divide one side (only) by a factor cos$x$ or $\cos^2 x$. However, this question provided the majority of candidates with three or four straightforward marks.

Answer: 9.6°, 90°.

Question 4

This question proved immensely popular, and was generally successfully attempted. Where marks were lost, this was due to sign errors or to putting f(–1) = 0, rather than –6. More seriously, some candidates set f(–2) = 0 and f(+1) = –6.

Answer: a = 1, b = 2.

Question 5

(i) Barely half of all candidates could successfully differentiate $y = x^2 \ln x$ and them simplify the derivative as $x(2\ln x + 1) = 0$. Thus few saw that $\ln x = -\frac{1}{2}$ is the key result when y = 0, and hence $x = \mathrm{e}^{-\frac{1}{2}}$ at the stationery point.

(ii) Because so many initially correct first derivatives had been poorly simplified, few correct second derivatives were later forthcoming, but Examiners gave credit for conceptually sound reasoning. Because of the nature of $\frac{\mathrm{d}y}{\mathrm{d}x}$, it was not simple to argue from values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at values of x just below and just above $x = \mathrm{e}^{-\frac{1}{2}}$, though a few candidates managed to do so from values x = 0, 1.

Answers: **(i)** $\mathrm{e}^{-\frac{1}{2}}$; **(ii)** Minimum point.

Question 6

(i) Few recognisable graphs of y = cot$x$ were in evidence, with the key features that $y \to \infty$ as $x \to 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$ for $0 < x < \frac{1}{2}\pi$, and $y\left(\frac{\pi}{2}\right) = 0$ not understood.

Several candidates got round their confusion concerning y = cot$x$ by correctly stating that cot$x$ = x is equivalent to saying that cos$x$ = $x$sin$x$ or that tan$x$ = $\frac{1}{x}$; they then produced excellent graphs of their left- and right-hand sides.

(ii) A large proportion of candidates still do not appreciate that cot$x$ = x means that f(x) ≡ cot$x$ – x = 0, and that once they show that f(0.8) and f(0.9) differ in sign then the proposition is proved. Equally well, use can be made of $f_1(x) \equiv x - \cot x$, $f_2(x) \equiv \cos x - x\sin x$, $f_3(x) \equiv \tan x - \frac{1}{x}$, etc.

(iii) Most solutions were wrongly based on use of x = 0.8, 0.9. One simply takes the tangent of each side of the equation given in part **(iii)**, or re-writes the equation of part **(i)** as $\tan x = \frac{1}{x}$ which implies that $x = \tan^{-1}\left(\frac{1}{x}\right)$.

(iv) There were still some candidates taking x = 0.8 (or 0.9) as being measured in *degrees*, though this error is much less evident than in years past. The Examiners were impressed by the good iteration generally seen, but again would urge candidates to work to 4 decimal places when iterating towards a root correct to 2 decimal places, i.e. in general, to 2 decimal places more than is required for the final degree of accuracy.

Answer: **(iv)** 0.86.

Question 7

Part **(i)** was well attempted. In parts **(ii)** and **(iii)**, there were problems with the differentiation and integration of exponential functions, with the key results $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\lambda x}\right) = \lambda \mathrm{e}^{\lambda x}$ and $\int \mathrm{e}^{kx}\mathrm{d}x = \frac{1}{k}\mathrm{e}^{kx} + C$, where λ and k are constants, not widely seen.

In part **(ii)**, the final mark was often lost by omitting to find where the tangent crosses the x-axis. For the definite integration of part **(iii)**, most candidates made a reasonable attempt and realised the importance of correct use of the lower limit x = 0.

Answers: **(i)** (0, 5); **(ii)** $y = 5 - 4x$, (1.25, 0); **(iii)** 4.7.

Question 8

(i) Too often an approximate value of R was given, rather than the exact one, and many believed that $R = \sqrt{1^2 + 1^2}$ meant that R = 1. Many left α in degrees or inexact radian values rather than the exact radian value required.

(ii) The result could not be obtained if R and/or α were incorrect in part **(i)**. Other solutions involved convoluted attempts to expand the denominator on the left-hand side.

(iii) Most solutions were excellent. A few used an incorrect quotient formula or said that $\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = +\sin x$.

(iv) Only around half of the solutions used the results from parts **(ii)** and **(iii)**, as instructed. Many used $\int \frac{\mathrm{d}\theta}{\sec^2\left(\theta - \pi/4\right)}$ or expanded the denominator as 1 + sin2θ, which they were unable to integrate. Use of $\sec^2\left(\theta - \pi/4\right) \equiv \left(\sec^2\theta - \sec^2\left(\pi/4\right)\right)$ was not infrequent.

Answer: **(i)** $R = \sqrt{2}$, $\alpha = \frac{\pi}{4}$.

Papers 8719/03 and 9709/03

Paper 3

General comments

There was wide variation in the standard of work by candidates on this paper and a corresponding range of marks from zero to full marks. All the questions appeared to be accessible to candidates who were fully prepared and no question seemed to be of exceptional difficulty. However, in the case of **Question 3** (algebra) completely satisfactory solutions to the final part were rare.

Adequately prepared candidates appeared to have sufficient time to attempt all questions.

Overall, the least well answered questions were **Question 5** (iteration), **Question 9** (vector geometry) and **Question 10** (differential equation). By contrast, **Question 1** (binominal expansion), **Question 4** (trigonometrical identity and equation) and **Question 8** (partial fractions) were felt to have been answered well.

The detailed comments that follow inevitable refer to mistakes or misconceptions and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts showing very good and occasionally excellent understanding of all the topics being tested.

Where numerical and other answers are given after the **Comments on specific questions**, it should be understood that alternative forms are often possible and that the form given is not necessarily the only 'correct' answer.

Comments on specific questions

Question 1

This question was generally answered well. Some candidates expanded $(2 + x)^{-3}$ directly, but the majority took out the factor of 2^{-3} and expanded $\left(1+\frac{1}{2}x\right)^{-3}$. The main sources of error were incorrect factors, e.g. 2 or $\frac{1}{2}$, and failure to obtain the correct coefficients of x^2.

Answer: $\frac{1}{8}-\frac{3}{16}x+\frac{3}{16}x^2$.

Question 2

Though many candidates completed this question confidently and accurately, others showed poor understanding of the logarithmic and exponential function and made little or no creditable progress.

Answer: 0.58.

Question 3

The first two parts of this question were usually answered well. However, it was clear that having factorised $p(x)$ correctly as $(x - 2)(2x^2 + x + 2)$, most candidates lacked a complete strategy for solving the inequality $p(x) > 0$. It was not uncommon for candidates to establish that $2x^2 + x + 2$ was never zero but almost all failed to show that it was positive for all values of x.

Answers: **(i)** -3; **(ii)** $(x - 2)(2x^2 + x + 2)$; **(iii)** $x > 2$.

Question 4

There were a pleasing number of successful proofs of the identity in the first part of this question. A common error was to apply the factor of 2 to both the numerator and the denominator of the right hand side.

The second part was often answered correctly though premature approximation caused some candidates to lose one, and sometimes two, of the accuracy marks for the angles.

Answer: **(ii)** 9.7°, 80.3°.

Question 5

Many candidates were unable to express the area of triangle *ONB* correctly in terms of r and α. Those who succeeded in doing so and equated the expression to one half (and not twice) the area of the sector *OAB*, usually went on to obtain the given relation correctly.

In part **(ii)** most candidates tried to sketch $y = x$ and $y = \sin 2x$. Many sketches were faulty and even when correct some candidates failed to make clear the connection between the presence of the intersection and the existence of a root in the given range.

Most, but by no means all, candidates showed the ability to use an iterative formula and find a root to some given degree of accuracy. The most common error here was to carry out the calculations with the calculator in degree mode rather than in radian mode.

Answer: **(iii)** 0.95.

Question 6

In the first two parts most candidates had a sound method for finding $\frac{u}{v}$ and its argument. However arithmetic errors were quite common. In addition there were surprisingly many errors in finding $u - v$, the incorrect answer $-3 + 5\mathrm{i}$ occurring quite frequently. Candidates seemed to find part **(iii)** difficult and usually gave incomplete or incorrect answers. Examiners found that satisfactory exact proofs of the given result in part **(iv)** were rare.

Answers: **(i)** $-3 + \mathrm{i}$, $\frac{1}{2} + \frac{1}{2}\mathrm{i}$; **(ii)** $\frac{1}{4}\pi$; **(iii)** *OC* and *BA* are equal and parallel.

Question 7

The first part was answered well by many candidates, though some found it hard to solve the equation obtained by equating the first derivative to zero. The integral in the second part was often attempted in the appropriate manner. The main sources of error were incorrect division by $-\frac{1}{2}$ when integrating $e^{-\frac{1}{2}x}$, and failure to keep a careful check on the work for errors of sign.

Answers: **(i)** 4; **(ii)** $16 - 26e^{-\frac{1}{2}}$.

Question 8

This was the best answered question on the paper, and there were many fully correct solutions. The most common difficulty in part **(b)** was in dealing with the logarithms at the end. In addition Examiners expected but did not always find a complete and full justification of the given answer.

Answer: **(a)(i)** $\frac{A}{x+4} + \frac{Bx+C}{x^2+3}$; **(ii)** $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ or $\frac{A}{x-2} + \frac{Bx+C}{(x+2)^2}$.

Question 9

Most candidates had a complete strategy for the problem in part **(i)** and set their work out clearly. However, there were many arithmetic and transcription errors. The importance of checking work cannot be over-emphasised. The work on part **(ii)** was disappointing, many candidates lacking a valid method. Those who had a sound method, usually equating the scalar product of $\overrightarrow{PQ}$ with a direction vector for l to zero and solving the resultant linear equation in *s*, often made sign errors in obtaining their expression for $\overrightarrow{PQ}$ in terms of *s*.

Answer: **(ii)** 4**i** + **j** + 2**k**.

Question 10

In part **(i)** only a few candidates could give a correct justification of the given differential equation. In part **(ii)** most candidates separated variables correctly but there were many poor attempts at finding the integral of $\frac{x-3}{x}$. Also some candidates failed to complete this part by rearranging their expression to give *t* in terms of *x*. In part **(iii)** Examiners found that more candidates made the substitution $x = 4$ rather than the correct one $x = 1$.

Answers: **(ii)** $t = 200\,(x - 3 - 3\ln x + 3\ln 3)$; **(iii)** 259 s.

Paper 9709/04

Paper 4

General comments

Most candidates scored well in the first three questions, very few scoring fewer than 8 of the 15 marks available. **Question 7** was also a fruitful source of marks.

The most usual places where very good candidates failed to score full marks were in **Question 4 (iv)** and **Question 6 (ii)**. In **Question 4 (iv)** even very good candidates made the implicit and erroneous assumption that the acceleration is constant. This is an error that has been made with considerable frequency in corresponding questions at previous sittings of this paper, and is less understandable on this occasion because the question says explicitly that 'the driving force is not now constant'. In **Question 6 (ii)** few candidates were able to show satisfactorily that $a \leqslant 4$, but for good candidates this was not a barrier to obtaining the maximum value of *P*.

Premature approximation was not in general a significant feature of candidates' work on this occasion. However, premature approximation was responsible for many candidates failing to score both of the marks available, under a special ruling, in the case where candidates took the acceleration to be constant in **Question 4 (iv)**. Only one of the two marks could be scored when a value for the acceleration was used that was insufficiently accurate to give the answer for the work done to the required degree of accuracy. In **Question 5 (i)** candidates often failed to score the final A mark following through. This is because the incorrect value of *t* was prematurely approximated, and thus did not admit a follow through answer for the speed to a sufficient degree of accuracy.

Comments on specific questions

Question 1

Candidates who realised that the only horizontal force acting on *P* is the tension in the string were usually successful in applying Newton's second law correctly to both particles, and subsequently in solving the resultant equations correctly for *a* and *T*.

However too many candidates sought to use a recipe rather than Mechanics principles, often resulting in a weight term appearing in the Newton's law equation for *P*, or the misapplication of $m_Q g = (m_P + m_Q)a$, which is the correct formula arising from eliminating *T* from the two relevant Newton's law equations.

Answer: 1.5 ms^{-2}, 2.55 N.

Question 2

Most candidates scored both marks in part **(i)**, and those who resolved forces parallel to the plane were usually successful in scoring all three marks in part **(ii)**. However, some candidates effectively took P N as the horizontal component of the applied force in **(ii)**. This applied force was taken to be parallel to the plane. Accordingly such candidates wrote $P \div \cos 30^\circ$, instead of $P \times \cos 30^\circ$, equal to $18\sin 30^\circ$.

Many candidates resolved forces in directions other than parallel to the plane, often without including the normal reaction force.

Answers: **(i)** $P = 9$; **(ii)** $P = 10.4$.

Question 3

Almost all candidates recognised the need to apply Newton's second law, although a significant minority just resolved forces down the plane, omitting the *ma* term. The most common errors were of sign, and of omission of the weight component.

Answer: 30.3 ms^{-1}.

Question 4

Almost all candidates answered parts **(i)** and **(ii)** correctly, but very few gave the correct answer for part **(iii)**. Frequent wrong answers in part **(iii)** were 5000 kJ and 2400 kJ, the former arising from the incorrect assumption that the resistance is equal to the driving force, and the latter arising from the incorrect assumption that the required work done is the gain in potential energy minus the kinetic energy. This kinetic energy is of course constant because the speed is constant, but it seems likely that such candidates were thinking of a gain in kinetic energy after implicitly and incorrectly assuming that the lorry starts from rest at the bottom of the hill.

In part **(iv)** almost all candidates assumed incorrectly that the acceleration is constant, thus limiting their mark to a maximum of 2 out of 5 under a special ruling in the mark scheme.

Answers: **(i)** 3200 kJ; **(ii)** 5000 kJ; **(iii)** 1800 kJ; **(iv)** 7200 kJ.

Question 5

Many candidates scored the method mark available in part **(i)** and although the majority proceeded to find the correct value of v_P, there were errors in sign in some other cases. It was also common for candidates to have the factor 1.8 the wrong way round. Some candidates set up and solved a correct equation for v_Q, but did not then proceed to an answer for v_P.

Some candidates used $v^2 = u^2 + 2as$ for P and Q separately, and assumed that s is the same for both. Such candidates failed to score the method mark.

Many candidates used an appropriate method for setting up an equation for t in part **(ii)**, but again sign errors were common. There were also very many answers in which candidates set up two quadratic equations in t, following the incorrect assumption that the distance travelled by each of P and Q is 51 m. Such candidates scored no marks in part **(ii)**. Candidates who pursued this erroneous method to the point of finding the positive root of each of the two equations usually gave their answer as the difference in these positive roots.

There was a significant minority of candidates who sought to apply the principle of conservation of momentum to part **(ii)**. This seems strange, not only because of the irrelevance of momentum, but also because the topic of momentum is not included in the syllabus for 9709.

Answers: **(i)** 9 ms^{-1}; **(ii)** 3 s.

Question 6

In part **(i)** the two relevant relationships can be summarised as $P = F \leqslant \mu R$, and in the first stage of part **(ii)** the two relevant relationships can be summarised as $ma = F \leqslant \mu R$. However, candidates more often wrote $P \leqslant F = \mu R$ and $ma \leqslant F = \mu R$ respectively. If F represents the frictional force then clearly the latter is incorrect. Nevertheless if there was any suggestion in the candidate's work, no matter how tenuous, that F represents the maximum possible value of the frictional force, Examiners gave candidates the benefit of the doubt and interpreted F in this way. Candidates should be encouraged to write precise mathematical statements, to avoid putting at risk marks that are within their grasp.

Very many candidates obtained μR as $0.75 \times 8000 = 6000$ in part **(i)**, but a significantly fewer number were able to satisfactorily complete the argument that the boxes remain at rest if $P \leqslant 6000$.

The first stage of part **(ii)** was poorly attempted and few candidates recognised the need to apply Newton's second law to either the upper box, or to both the lower box and the combined boxes. Most candidates who applied Newton's second law to the upper box found F_{max} as 0.4×4000, although some used the incorrect 0.4×8000. Another common error was to introduce P into the equation and this meant that candidates could not produce an inequality for a by considering the upper box only. Frequently the value of 6000 for P was brought forward from part **(i)** into an equation or equations for the first stage of part **(ii)**.

Most candidates who were successful in obtaining the maximum value of P applied Newton's second law, with $a = 4$, to the combined boxes rather than to the lower box.

A circular argument in which $a = 4$ was used to find P_{max}, and then this value used to find a_{max}, was common.

Answer: **(ii)** $P_{max} = 9200$ N.

Question 7

In part **(i)** almost all candidates attempted to find the non-zero root of $v = 0$, although a few obtained this root incorrectly as 9 or $\sqrt{90}$. Some candidates attempted to solve $\frac{dv}{dt} = 0$.

Part **(ii)** was generally well attempted although there was a significant number of candidates who attempted to use formulae applicable only to motion with constant acceleration.

Part **(iii)** was very well attempted, with many candidates who performed poorly in part **(ii)** scoring all four marks here. It was clear that this is because the exercise was treated as one of Pure Mathematics (find the maximum value of the given function $v(t)$), including the verification that the value is indeed a maximum and not a minimum. This verification is not necessary in the context of the actual question, since it is clear from the answers in parts **(i)** and **(ii)** that v increases its speed from zero to a maximum and then decreases its speed to zero during the 90 s interval.

Answers: **(i)** 90 s; **(ii)** 547 m; **(iii)** 10.8 ms^{-1}.

Papers 8719/05 and 9709/05
Paper 5

General comments

With the exception of **Question 7 (ii)**, all candidates, except the very weakest, found that they could make some progress with all the questions on the paper. With few exceptions, candidates had sufficient time to attempt all the questions. On the whole solutions were well presented with candidates invariably attempting to state which mechanical ideas were being applied in order to solve the question.

A number of candidates had poor attention to detail. The instructions on the front page of the question paper clearly state, 'Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees'. For example the answer to **Question 7 (i)** was often given as 0.17 m rather than 0.171 m. It should also be borne in mind that working to 3 significant figures with intermediate answers does not necessarily guarantee that the final answer will be correct to this degree of accuracy. For example in **Question 4 (ii)(a)**, if the angle θ is taken to be 36.9°, the height obtained is 45.1 m rather than 45 m. In all calculations candidates should work with the best values given by their calculators and then round the final answer only to the required degree of accuracy.

Comments on specific questions

Question 1

Competent candidates scored well on this question. Many of the less able, however, made the frequent error of assuming that the tension in the string retained its original value of 20 N when the particle was suspended in the equilibrium position. This error betrayed a complete ignorance of the understanding of Hooke's Law in that those candidates failed to realise that if the extension of the string increased then so must the tension in it.

A number of candidates across the ability range misinterpreted the question and tried to apply the conservation of energy principle in order to find W. This approach wrongly assumed that the particle was released from rest at the level of AB, and then found the value of W which would then cause the particle to come to its next position of rest at a distance 0.75 m below the level of AB.

Answers: **(i)** 20 N; **(ii)** $W = 48$.

Question 2

There seems to have been an all round improvement in the response of the candidates to statics questions. In this question, only the weakest candidates failed to realise that it was necessary to take moments about A in order to find the tension in the string. Generally this question was well done, the main failing being to assume that the vertical component of the force at A was equal to the vertical component of the tension in the string.

Answers: **(i)** 39 N; **(ii)** 36 N and 15 N.

Question 3

Part **(i)** of this question was very well answered by practically all candidates.

Able candidates coped well with solving the differential equation but some made extra work for themselves by quoting $t = 0$ when $v = 0$ as the initial condition to find the constant of integration rather than taking the more obvious $t = 0$ when $v = 10\ \text{ms}^{-1}$.

Other attempts were marred by poor algebraic manipulation when separating the variables prior to solving the differential equation. Inevitably many thought that the problem could be solved by using the constant acceleration equations of motion. In this question the driving force of the car varied with its speed and hence, as the resultant force on the car varied during the motion, so also must the acceleration. Thus equations such as $v = u + at$ could not be used to solve this sort of question.

Answer: **(ii)** 30.6 seconds.

Question 4

Candidates of all abilities found this to be a very accessible question, with many scoring full marks.

Some of the explanations for part **(i)** were a bit vague. The bare statement $\cos\theta = \frac{40}{50}$ with no explanation or supporting diagram was not considered to be a sufficient explanation.

Of the many ways of attempting part **(ii)(a)** the most popular was substituting into $v^2 = u^2 - 2as$ with $v = 0$. A perplexing frequent error was that a number of candidates quoted this formula correctly but then omitted to square $u = 50\sin\theta$ when substituting. The most frequent error in part **(ii)(b)** was to find the time taken to reach the highest point of the path (3s), but then omitted to double it to find the range on the plane.

A few candidates who had little idea of considering components of the velocity apparently obtained the correct answer by stating $40^2 = 50^2 - 2gH$. Provided that the candidate made it clear that this equation had been derived from a consideration of the conservation of energy principle full credit was allowed. These candidates usually betrayed their ignorance later by stating $40 = 50 - gT$.

Answers: **(ii)(a)** 45 m, **(b)** 240 m.

Question 5

Nearly all candidates obtained the loss in E.P.E. correctly and many found the value of the frictional force (F = 1.6 N), but only the better candidates were able to make further progress. The work done against friction often did not appear in the energy equation and, if it did, F was often multiplied by 0.5 rather than 0.1. Another failure in establishing the energy equation was to introduce an incorrect extra term. The candidates who did this first found the initial tension in the string (T = 4 N) and then stated that the work done was (4 – 1.6) x 0.1. This was a double error in that the tension was not constant in the subsequent motion and the energy associated with the tension in the string had already been accounted for with the E.P.E..

It was surprising how many candidates read the question carelessly and had a situation in which the particle was initially 0.5 m vertically below the point O.

Answer: 0.316 ms^{-1}.

Question 6

There was a very good response to the first part of this question and only a small proportion of the candidates failed to find the coefficient of friction correctly.

In part **(ii)**, able candidates experienced little difficulty as they appreciated that the first step in the solution was to find the new value of OP. The attempts of the remainder were disappointing in that it was assumed that OP was still 0.5 m. Many attempts were simply $w = \frac{1.2}{0.5} = 2.4$ rad s^{-1}. A moment's pause should have alerted candidates to think a little more about the question, as such a simple statement was hardly going to merit the award of 5 marks. Other attempts correctly identified that the value of the limiting friction found in part **(i)** was unaltered, but then OP = 0.5 was used to find a different value of the coefficient of friction. This was not acceptable as the question made it clear that it was the same particle on the same turntable.

Answers: **(i)** 0.45; **(ii)** 3.75 rad s^{-1}.

Question 7

In part **(i)** the general idea of taking moments about AB was well understood. Sometimes the height of the centre of mass of the grain was taken to be 0.1 m above AB, but the usual errors were with arithmetical carelessness in calculating the areas of the rectangle and triangle. In some solutions there was much wastage of time in unnecessarily finding the distance of the centre of mass of the grain from AD. The second part of the question was not well answered. To make any progress it was necessary to find the areas as functions of y and also to appreciate that the centre of mass of the grain was vertically above B. In the majority of cases the areas were taken to be the same as those found in part **(i)** of the question. For those abler candidates who made some headway with the problem, the most frequent error occurred when moments were taken about AD. It was realised that the centre of mass of the grain was 0.4 m from AD, but the moment of the triangle was taken to be $y^2 \times \frac{2y}{3}$ rather than $y^2 \times \left(\frac{2y}{3} + 0.4\right)$.

Answers: **(i)** 0.171 m; **(ii)** 0.346.

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks. The standard was high with many candidates only doing poorly on **Question 6**, which did not seem to be a familiar situation. Work was clearly presented and there were relatively few premature approximations or answers to 2 significant figures.

Comments on specific questions

Question 1

There were many answers of 9! which indicated that repeated letters had not been considered. Correct answers to part **(ii)** were not very common, but seeing 5! or 4! implying some arrangement of either vowels or consonants gained one mark, irrespective of what else was considered.

Answers: **(i)** 90 720; **(ii)** 720.

Question 2

(i) This was usually correct.

(ii) This was well drawn. Common mistakes were starting the vertical scale at a non-zero number, and having non-linear scales.

(iii) This was mainly correct with a few candidates using frequency density rather than frequency.

Answers: **(i)** 40; **(iii)** $\frac{60}{68}$ or 0.882.

Question 3

This question was very well done by most candidates. A few did not appreciate the conditional probability situation in part **(ii)** but overall this was a straightforward question and candidates who had prepared for this gained full marks.

Answers: **(i)** 0.072; **(ii)** 0.25.

Question 4

A surprising number of candidates lost a mark in the calculation of the standard deviation in part **(i)** due to premature approximation of the mean from 41.39 to 41.4. Most managed to find the age of the person who left the group. Fewer candidates coped with finding the standard deviation of the remaining people.

Answers: **(i)** 13.2; **(ii)** 48, 13.4.

Question 5

In part **(i)** candidates were not making use of the tables at the foot of the page of normal tables, and thus some lost an accuracy mark. As usual a wide and varied list of *z*-values appeared, many of them Φ-values by mistake. These gained no marks for part **(i)**. The second part discriminated between those candidates who read the question carefully and those who did not. Almost everybody except the highest grade candidates just did not read the question carefully '….on every one of the next four days,' and stopped after having found the initial probability, and did not raise it to the power 4. A few multiplied by 4 and some divided by 4.

Answers: **(i)** 48.6; **(ii)** 0.00438.

Question 6

(i) Some candidates attempted to answer this by using combinations instead of listing outcomes. Others listed 60, or even 125, outcomes, which must have been very time consuming.

(ii)(iii)(iv) These parts depended to some extent on part **(i)**. If part **(i)** was correctly answered, then many candidates gained full marks. Most were able to gain some method marks for evaluating the expectation and variance of their random variable, provided that their probabilities summed to 1, even if part **(i)** was incorrectly answered.

Answers: **(i)** 0.4; **(ii)** 0.3; **(iii)** P(3) = 0.1, P(4) = 0.3, P(5) = 0.6; **(iv)** E(L) = 4.5, Var (L) = 0.45.

Question 7

Part **(i)** was clearly quite difficult for some candidates. The marking was generous with 'item', 'term' and some other alternative forms for 'trial' being accepted. Many candidates gave the conditions for which a binomial distribution may be approximated by the normal, which of course, gained no marks. The rest of the question was generally very well answered.

Answers: **(ii)** 0.419; **(iii)** 0.0782.

Paper 8719/07 and 9709/07

Paper 7

General comments

Overall, this proved to be a fair and testing paper. There was a good spread of marks, with many in the range twenty to forty, and it was pleasing to note that there were very few candidates who appeared to be totally unprepared for the examination. **Question 6** was particularly well attempted, even by the weakest candidates, with calculus skills being notably well developed, whilst **Question 7** proved to be a very good discriminator. Further comments on **Question 7** are made below, but very few candidates scored highly here, with the majority of candidates, including very good candidates, only scoring about half marks.

Numerical work and levels of accuracy shown were very good. The only questions where accuracy was an issue were **Question 2** and **Question 5**. In **Question 2** premature rounding of z values or Φ-values could have led to a final answer that was not correct to three significant figures and in **Question 5 (ii)** early calculation of $\frac{1499}{1500}$ led to large errors in n. It was pleasing to note that the majority of candidates successfully gave the required three significant figures accuracy, or better, in final answers.

In general, most candidates were able to complete the paper and lack of time did not appear to be a factor. Work was usually well presented with method and working clearly shown.

Detailed comments on individual questions are as follows, though it should be noted that whilst the comments below indicate particular errors and misconceptions, there were also many very good and complete answers to each question.

Comments on specific questions

Question 1

A minority of candidates made no attempt at this question, not recognising that a Poisson Distribution was required. For those who successfully attempted a Poisson Distribution, common errors were to work with the wrong mean (1.8, 2.3 or 5.1 were commonly seen, rather than the correct mean of 6.9). A failure to sum P(6) + P(7) + P(8) was also noted in a few cases. A few candidates stated Po(6.9) but unfortunately went no further.

Many candidates scored full marks.

Answer: 0.428.

Question 2

(i) Most candidates attempted to standardise, though the most common error was failure to work with $\frac{\sigma}{\sqrt{n}}$. Some candidates lost the final accuracy mark due to premature rounding at earlier stages (as mentioned above), but on the whole this part was well answered.

(ii) This part was not well answered, despite the fact that many candidates stated in part **(i)** that they were applying the Central Limit Theorem (CLT). The impression was given that a set method was being applied in part **(i)**, but candidates' answers to part **(ii)** displayed a lack of understanding of what they were actually doing. Answers included references to 'the weather…' (being 'predictable..') or 'the mean and/or standard deviation..' (being 'known…'). There were very few references here to the CLT, so very few candidates gained this mark.

Answers: **(i)** 0.580; **(ii)** 300 is large, so CLT can be applied.

Question 3

(i) There were many candidates who achieved full marks. Most candidates successfully found the mean, but a few errors were noted in finding the variance. These included substituting incorrect values into an initially correct formula (mainly caused by confusion between $\frac{\left(\sum t\right)^2}{n}$ and $\left(\frac{\sum t}{n}\right)^2$, and in a few cases candidates calculated the biased variance rather than unbiased.

(ii) Calculation of the confidence interval was well attempted in part **(ii)**, with candidates showing familiarity with its form. Errors included a wrong z-value (with 1.555 being the most commonly used incorrect value), use of a Φ-value instead of a z-value (0.8340), and omission of square roots in the formula.

Answers: **(i)** 27.2, 324; **(ii)** (24.4, 30.0).

Question 4

Candidates were given a good start to this question by being asked to *show* that E(X) was 469, and Var(X) was 295. This helped to avert errors in part **(ii)**, resulting in a reasonably well attempted question. There were, however, candidates who were unable to successfully show that the variance in part **(i)** was 295, and used a value of 195 or similar throughout the question, along with incorrect values for the variance of Y and $X - Y$. Some method marks and follow through marks were available, but candidates are advised to use the values given rather than their own incorrectly calculated ones in such a case. Calculation of the required means did not cause many problems.

The most common loss of one mark on this question was caused by candidates' failure to read the question fully. The question clearly asked for the mean and *standard deviation* of $X - Y$. The majority of candidates merely gave the mean and variance.

On the whole this was a well attempted question.

Answers: **(ii)** 14, 23.8, 0.369.

Question 5

(i) Many candidates realised that a Poisson Distribution was required here. Some candidates attempted a Normal Distribution, which was not a suitable approximation, and some candidates did not use any distributional approximation at all, but merely calculated the probability using the original Binomial Distribution. Marks for use of Normal or Binomial were limited, as this was not what was required by the question. The candidates who successfully chose to use a Poisson Distribution usually did well and were able to reach the correct answer. Errors included use of an incorrect mean, and calculation of 1 – P(0) – P(1), rather than 1 – P(0) – P(1) – P(2).

(ii) Candidates could have attempted this part of the question either by using a Poisson or a Binomial Distribution. Both of these methods involved solving an equation where *n* was a power, and it was pleasing to note that candidates were generally successful in their solutions of this equation. As mentioned previously, accuracy became an issue for candidates who used the Binomial approach and calculated $\frac{1499}{1500}$ at an early stage and approximated prematurely to 0.999.

Answers: **(i)** 0.269; **(ii)** 6908 or 6906.

Question 6

This was a very well attempted question. However, some errors were noted including incorrect limits in part **(i)** (0 to 0.5 or 0 to 1, or in a few cases 0 to 3 with a loss of the factor of 3 in the function, were seen), and careless numerical slips. A common error in part **(ii)** was to merely calculate $E(X^2)$, quoting this as the answer for the variance, without subtracting the value of the mean squared. Algebraic errors were also seen, mainly in the removal of brackets, but on the whole attempts at integration were very good. A few candidates confused the mean with the median.

Answers: **(i)** 0.125; **(ii)** 0.25, 0.0375.

Question 7

Whist this was not, in itself, a particularly difficult question, it proved problematic for many candidates. A large number of candidates tried to approximate to a Normal Distribution, rather than using a Binomial Distribution, and seemed unfamiliar with hypothesis testing in the context of a discrete random variable.

(i) There were many inadequate responses; merely re-stating what was given in the question was not sufficient (e.g. many candidates just said 'he only took plants from the front row', they did not say why this was not appropriate). To say that he should have taken plants from other rows was still not fully appreciating the idea that all plants should have equal chance of being selected in a random sample. Candidates who commented on the fact that conditions may be different on the front row and so the sample was not representative, clearly appreciated what was required and were awarded full credit.

(ii) This part caused considerable problems. Most candidates correctly stated that a one-tailed test was required, and many set up a correct null and alternative hypothesis. However, the majority of candidates were unable to find the critical region, and even the method for doing this correctly was seldom seen. Some candidates found a critical region (which was occasionally correct), but did not show how this had been obtained, whilst other candidates found a correct region by an incorrect method (e.g. comparing $P(X = 5)$, and $P(X = 6)$ etc. with 0.5). It is very important that full working and all comparisons are shown in this type of question as correct answers may be obtained by fortuitous means. The majority of candidates carried out the test without actually finding the critical region, by comparing $P(X \geqslant 4)$ with 0.5, and full credit was given for this when correctly done.

(iii) Answers here were seldom related to the question, with the majority of candidates merely quoting a text book definition. This did not gain any marks, as an answer relating to the context of the question was required.

(iv) Candidates who found the correct critical region usually went on to answer this correctly.

Answers: **(i)** Not random, could be more light, etc.; **(ii)** One-tailed test, H_0: $p = 0.35$, H_1: $p > 0.35$, Critical region is 6,7,8 survive. No significant improvement in survival rate; **(iii)** Saying no improvement when there is; **(iv)** 0.950.